Low speed and medium speed theories can be obtained as special cases of Equations (7) and (8).

The above new theories are ready for testing as soon as suitable continuous withdrawal data becomes available.

DISCUSSION

Equations (7) and (8) are new theories and the forms of Equations (3), (4), and (5) are new.

The derivation of the suitable dimensionless form of Equation (4) suggested the method of addition for deriving new withdrawal theories and led to the understanding of the method. The suitable dimensionless form simply requires that the equation be speed explicit on one side (such as C_o) and that the film thickness (h) be included only in the parameter D.

The speed-explicit form (2) was first developed in dimensional form for the flat plate Ellis theory equation. The original purpose for the development (2) was to simplify numerical evaluation of the functional relationship between h and u_w for a given fluid. The method of addition presented herein is an important but unexpected use of the speed-explicit form.

Although development of the method of addition was based on the recent development of the speed-explicit form of withdrawal equations, it was not until after the speed-explicit paper (2) had been accepted that the suitable dimensionless form [Equation (4)] of the Ellis theory was derived.

ACKNOWLEDGMENT

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NOTATION

A = fluid property and thickness, Equation (4d)
a_0 = Ellis low shear parameter ($1/\mu$), Equation (4a)
a_1 = Ellis parameter, Equation (4a)
B = fluid properties, Equation (4c)
C_m = meniscus curvature, Equation (6d)
C_o = withdrawal speed parameter, $u_w(\mu/\sigma)$ or $u_w/(a_0\sigma)$
D = film thickness parameter, $h(\rho g/\sigma)^{1/2}$
G = radius parameter, $R(\rho g/2\sigma)^{1/2}$
g = acceleration of gravity, cm./sq.sec.
h = film thickness, cm.
P = curvature coefficient, Equation (5a)
R = cylinder radius, cm.
S = film thickness parameter, $1 + (h/R)$
u = fluid velocity, cm./sec.
u_w = withdrawal velocity of the solid, cm./sec.
Y = Y function, Equation (6c)
y = coordinate, cm.

Greek letters

α = Ellis exponent parameter, Equation (4a)
μ = viscosity, poise
ρ = density, g./cc.
σ = surface tension, dyne/cm.
τ = viscous shear stress, dyne/sq.cm.

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Some Properties of the Apparent Water Paradox in Entrainment

JOHN A. TALLMADGE and RICHARD STELLA

Drexel Institute of Technology, Philadelphia, Pennsylvania

Two entrainment phenomena of engineering importance are withdrawal and removal. We use withdrawal to mean the slow, vertical, upward passage of an axisymmetric solid through the free surface of a wetting liquid in such a way that liquid contact is maintained between the solid and the bath. Removal is a related condition in which a short object is raised above the bath after withdrawal so that the entrained film separates from the liquid bath. Removal experiments and continuous withdrawal experiments have been reported (*1, 7*) and discussed (*5*) elsewhere.

Substantial progress has been made in describing withdrawal. For example, a theory developed for the continuous withdrawal of cylinders has recently been verified using removal tests as well as continuous withdrawal experiments (*7*). Specifically, this theory was found valid for radii of 0.01 cm. to infinity and for a wide range of speeds. Furthermore, all fluids tested, except one, agreed with the theory. The exceptional fluid was water. The behavior of water was noted in the theory paper (*7*) but no data were given nor was any satisfactory explanation offered. The exceptional behavior of water was called the *water paradox* (*6, 7*); it might now be named the *apparent water paradox* in entrainment.

Richard Stella is with the Sun Oil Co., Toledo, Ohio.

It is the purpose of this note to describe some properties of this apparent paradox. The water used in the experiments noted below was distilled.

THE WATER PARADOX

Consider first the data which indicated the paradox. Although the theoretical removal mass per unit length (M/L) agreed with data within 5 to 20% for glycerine, motor oil, mineral oil, kerosene, and two mixtures of these fluids, the theoretical M/L for water (1.5 mg./cm.) was much lower than the M/L data for water (2.2 mg./cm.) at 1.59 cm./sec. withdrawal speed on a 0.346 cm. radius glass rod. At the higher speeds of 3.76, 7.56, and 11.5 cm./sec., the differences between theory (2.7, 4.3, and 5.7 mg./cm., respectively) and data (4.9, 8.2, and 11.5 mg./cm., respectively) were found to be larger. Thus it was concluded that the water data obtained in removal studies did not agree with theory.

The above water data were calculated by taking the slope of a removal mass (M) vs. immersion length (L) plot, as described previously (*6, 7*), and were obtained from results reported in thesis form (Runs 68, 69, 70, 23). The fluid properties and resultant parameters of Goucher number (0.902) and capillary numbers (0.00021 to 0.00149) needed to calculate theory M/L were obtained from values (*7*) in which minor typographical errors had been adjusted, as noted in the ADI tables of that reference.

The data of other fluids are compared with water results and with theory in Figure 1, by using coordinates suitable for comparing fluids. The flow thickness $\overline{h}$ was evaluated from data by $\overline{h} \doteq (M/L)/(\rho 2\pi R)$. The oil on brass data of Figure 1 are typical of the removal and withdrawal evidence which has been used to verify the theory. All the water on glass data (W) shown in Figure 1 are consistent with the M/L values noted above, including runs using deionized water, organic dye in aqueous solution, and water with an excess of surfactant (detergent) present. However, the values of the glycerine-water mixture on glass (GW) agree with theory.

In summary, Figure 1 describes the so-called "water paradox."

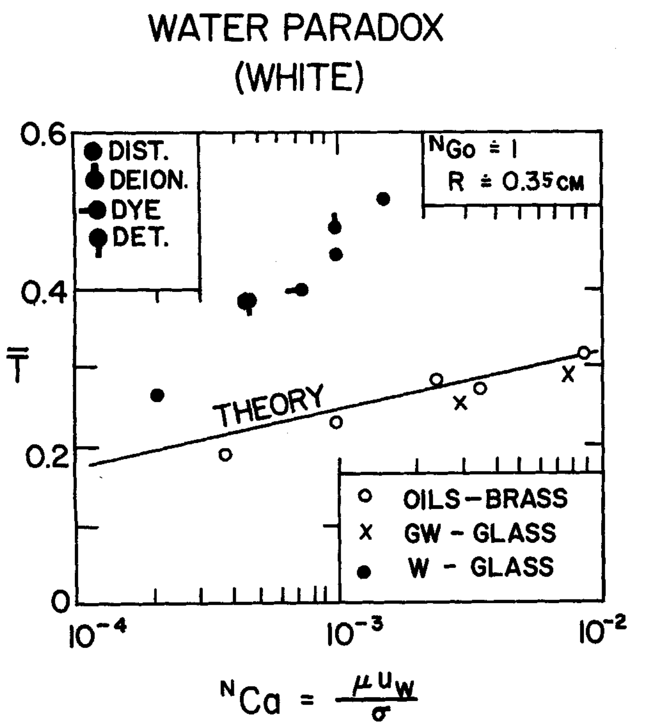

Fig. 1. The water paradox data of White (*6*).

ANALYSIS OF THE PARADOX

In order to determine factors which influence this paradox, the authors divided possible explanation into two groups: those mechanisms which influenced the gas-liquid interface (fluid category) and those which influenced the solid-liquid interface (solid category). Considered in the fluid category were the effect of air bubbles (*2*), pH (*2*), ionic concentration, nonpolar concentration, thin films, and liquid impurities. The mechanisms hypothesized in the solid category included those due to an adsorbed liquid layer, cleanliness (*3*), or surface roughness; the first is related to static holdup and the second is based on the initial condition of the solid.

Some of the possibilities were not tested in this work but are included to indicate the path of the investigation. In preliminary analyses reported earlier (*6, 7*), the following appeared to be inconsistent with the experimental facts: thin films, dynamic surface tension with impurities, and surface roughness.

The first exploratory tests involved the effect of ionic and nonpolar concentration, but did not resolve the paradox. Attention was then directed toward mechanisms of the solid category.

To influence the initial condition of the solid, one of the authors (*4*) varied the electrostatic charge on a clean glass rod by rubbing with rabbit fur. The amount of charge was found to affect the wetting junction and also the post-removal film distribution. This qualitative indication was followed by tests to determine the quantitative effect of varying the charge. Tests were made at 3.54 cm./sec. with a 0.37 cm. radius glass rod, or at $N_{Ca} = 0.00046$ and $N_{Go} = 1.02$; at these conditions the M/L value predicted by theory is 2.74 mg./cm. Data M/L were found to be 5.71 mg./cm. for no charge, 4.09 for a small charge, 3.22 for a medium charge, and 1.79 for a large charge, as noted in runs 3, 10, 14, and 16 of Stella (*4*). The data values show that changes in the initial condition of the solid substantially influenced the amount of removal mass.

The extent of the charge described above was based on visual observations. First, the medium charge was defined. The medium charge was the name given to the maximum charge which could be applied without changing the pre-separation film distribution. By definition, the large charge was any which was greater than the medium value. Thus the large charge was any amount which changed the film distribution before removal (or in other words, changed the wetting junction during withdrawal). The small charge was defined as any value intermediate between no charge and medium charge. The reason that the medium charge data agree with theory is not completely understood at this time, but may be fortuitous.

Figure 2 shows tests at other speeds which confirm the influence of charge on the removal mass. The no charge data of Figure 2 also confirm the speed effect and the magnitude of White's water data (Figure 1). In summary, the charge results clearly indicated that the past history of the solid is an important factor influencing removal mass.

Another pretreatment approach was studied to check the history effect: that involving chemical methods for cleaning. For comparison, we first describe the acid cleaning procedure used before each Figure 2 test. The acid cleaning method involved three steps: (a) immersing the cylinder in conventional cleaning solution (35 ml. of saturated sodium chromate per liter of concentrated sulphuric acid), (b) rinsing with cold tap water, and (c) washing with acetone. The no charge data of Figure 2 show that

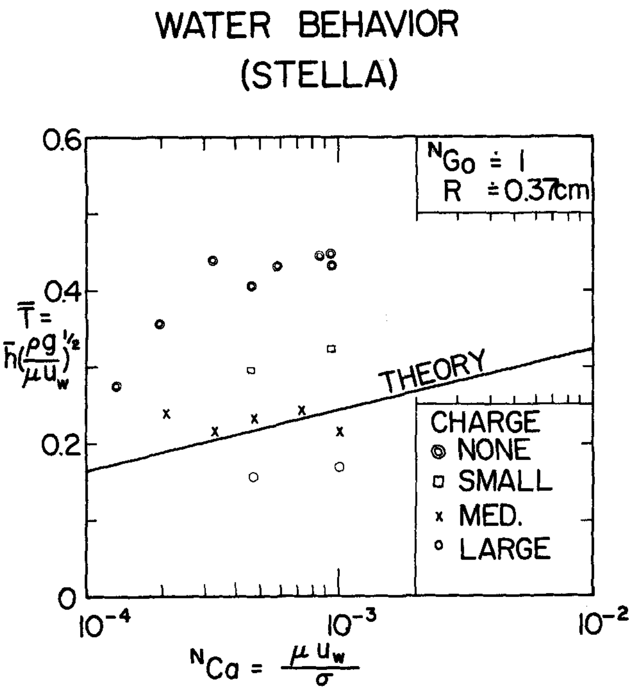

Fig. 2. The water behavior data of Stella *(4)*.

the acid pretreatment method resulted in water removal masses on glass which were considerably different from those of withdrawal theory.

Tests were made by comparing a second cleaning procedure (a simple acetone rinse) with the acid method, using the same speed and a stainless steel support. As expected, the acid cleaning resulted in a removal mass which was 35% larger than the theoretical value. By using only the acetone rinse for preparing the steel support, the removal mass was 15% below theoretical predictions. We conclude that the single change in the chemical pretreatment procedure resulted in a significant reduction of removal mass. Tests repeated at another speed confirmed the differences in removal mass and showed that the differences were similar to the electrostatic results of Figure 2.

The quantitative tests of Figure 2 were suggested by observations of film distributions in which motion of the wetting or dynamic junction was influenced by electrostatic charges. The wetting junction was influenced by the cleaning procedures in a similar way; the relationship between junction motion and flow thickness was similar. Based on the tests, the authors believe that pre-charge and pre-cleaning produce the same qualitative effects in entrainment and in junction motion.

Taken together, the cleaning and electrostatic results indicate that the primary reason for the paradox is the variable nature of the initial condition of the solid. Because the condition can be varied either by electrostatic charges or by different procedures of cleaning and chemical pretreatment, other factors which influence the initial condition of the solid may also be significant.

Based on this study of the properties of water removal, we conclude that:

1. It is not proper to compare the data from moving junction experiments with theories for systems which have no dynamic junctions present. Continuous withdrawal is a system with no dynamic junction. Thus removal tests with moving junction fluids (such as water) are not comparable with theoretical predictions for continuous withdrawal.

2. The water removal data give no information as to the expected performance of water in continuous withdrawal. It is not known how well or poorly water will agree with withdrawal theories such as that presented elsewhere (7). Unless evidence to the contrary is obtained, it is concluded that the cylinder withdrawal theories are valid for all fluids which have been properly tested. A proper test of water apparently will require a continuous withdrawal geometry.

3. The removal experiment with water shows that both drainage processes and film distribution processes in removal and unsteady withdrawal are considerably more complex than indicated by free drainage theories.

DISCUSSION

The original objective of this study was to determine whether water removal results indicated that continuous withdrawal theories did not apply to all fluids. In other words, is there a water paradox in continuous withdrawal? Within the limitation of removal tests, the original objective was met (see conclusion 2).

Although beyond the scope of this work, another question suggested by the results involves whether there is a water paradox in removal. Based on the data in Figure 1, it appears that water does behave differently from kerosene, mineral oil, and one glycerine solution. Based on observations of motion of the wetting junction, it also appears that water behaves differently, at least under the test conditions studied.

However, the question arises as to whether, in transient tests, changes in charge or cleaning noticeably affect the behavior of other fluids than water. This is an important question for future study. The resolution of the question apparently lies within the scope of removal, restricted drainage, and unsteady withdrawal problems (7).

ACKNOWLEDGMENT

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NOTATION

a = capillary length, $(2\sigma/\rho g)^{1/2}$, cm.
g = gravitational acceleration, cm./sq. sec.
$\bar{h}$ = flow thickness, cm.
L = length, cm.
M = removal mass, g.
N_{Ca} = capillary number, $\mu u_w/\sigma$, dimensionless
N_{Go} = Goucher number, R/a, dimensionless
R = radius, cm.
$\bar{T}$ = flow thickness, $\bar{h}$ $(\rho g/\mu u_w)^{1/2}$
u_w = withdrawal speed, cm./sec.

Greek letters

μ = viscosity, poise
ρ = density, g./cc.
σ = surface tension, dyne/cm.

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